

COMPLEX NUMBERS

SECTION 5.3

Lesson Objectives

Perform operations with complex numbers

- add
- subtract
- multiply
- divide

Operations with Complex Numbers

- Numbers that contain i , where $i = \sqrt{-1}$
- i is called imaginary
- Numbers that real and imaginary numbers are complex
- i is treated like any other square root
- Operations:
 - Addition/Subtraction
 - Multiplication/Division
 - Powers on i

Addition and Subtraction

- Add real parts and imaginary parts separately
- Examples:

$$\begin{aligned} & 3i + 4 - 2i + 9 - 8i - 1 \\ &= 4 + 9 - 1 + 3i - 2i - 8i \\ &= 12 - 7i \end{aligned}$$

$$\begin{aligned} & 3i - (4 - 2i + 9) - 6i + 2 \\ &= 3i - 4 + 2i - 9 - 6i + 2 \\ &= -4 - 9 + 2 + 3i + 2i - 6i \\ &= -11 - i \end{aligned}$$

$$\begin{aligned} & 2i - \{-(3+i) - 5i - (9 - 8i + 4)\} \\ &= 2i - \{-3 - i - 5i - 9 + 8i - 4\} \\ &= 2i - \{-16 + 2i\} \\ &= 2i + 16 - 2i \\ &= 16 \end{aligned}$$

$$\begin{aligned} & i + \{(4 - i) + 9 - (2i + 1)\} \\ &= i + \{4 - i + 9 - 2i - 1\} \\ &= i + \{12 - 3i\} \\ &= 12 - 2i \end{aligned}$$

Multiplication/Division

- Distribute terms (just like with other radicals)
- Since $i = \sqrt{-1}$, then $i^2 = (\sqrt{-1})^2 = -1$
- Examples

$$\begin{aligned} & -2i(3 - 4i) \\ &= -2i(3) - 2i(-4i) \\ &= -6i + 8(i^2) = -6i + 8(-1) \\ &= -6i - 8 = -8 - 6i \end{aligned}$$

$$\begin{aligned} & (2 - 3i)(1 - 4i) \\ &= 2(1) - 3i(1) + 2(-4i) - 3i(-4i) \\ &= 2 - 3i - 8i + 12(i^2) \\ &= 2 - 11i - 12 \\ &= -10 - 11i \end{aligned}$$

Multiplication/Division

➤ Multiply by "complex conjugate" to divide

➤ Examples

$$\begin{aligned} & \frac{(3-4i)}{-2i} \\ &= \frac{(3-4i) \cdot -2i}{-2i \cdot -2i} \\ &= \frac{-2i(3-4i)}{4i^2} \\ &= \frac{-6i + 8i^2}{-4} = \frac{-8-6i}{-4} \\ &= \frac{4+3i}{2} \end{aligned}$$

$$\begin{aligned} & \frac{(2-3i)}{1+i} \\ &= \frac{(2-3i) \cdot (1-i)}{(1+i) \cdot (1-i)} \\ &= \frac{(2-3i)(1-i)}{1-i^2} \\ &= \frac{2-3i-2i+3i^2}{1-i^2} \\ &= \frac{-1-5i}{2} \end{aligned}$$

Powers on i

➤ Cycle of 4 elements:

$$i = \sqrt{-1}$$

$$i^2 = (\sqrt{-1})^2 = -1$$

$$i^3 = (\sqrt{-1})^3 = -i$$

$$i^4 = (\sqrt{-1})^4 = 1$$

➤ All powers over 4 are one of these above:

$$i^{17} = i^{16}i = (i^4)^4i = 1(i) = i$$

$$i^{23} = i^{20}i^3 = (i^4)^5i^3 = 1(i^3) = -i$$

$$i^{56} = (i^4)^{14} = 1$$